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Non-abelian tensor-multiplet anomalies

Jeffrey A. Harvey

Enrico Fermi Institute and Department of Physics
University of Chicago, 5640 Ellis Avenue,
Chicago, IL 60637
E-mail: j-harvey@uchicago.edu

Ruben Minasian and Gregory Moore

Department of Physics, Yale University,
New Haven, CT 06520
E-mail: ruben@physics.yale.edu, moore@physics.yale.edu

Abstract: We use the anomaly cancellation of the M-theory fivebrane to derive the R-symmetry anomalies of the A_N $(0,2)$ tensor-multiplet theories. This result leads to a simple derivation of black hole entropy in $d = 4, \mathcal{N} = 2$ compactifications of M-theory. We also show how the formalism of normal bundle anomaly cancellation clarifies the Kaluza-Klein origin of Chern-Simons terms in gauged supergravity theories. The results imply the existence of interesting $1/N$ corrections in the AdS/CFT correspondence.

Keywords: M-Theory, String Duality, Black Holes in String Theory.

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Contents

1. Introduction

Anomalies are related to topology. Hence anomalous couplings are robust and serve as effective probes of the *terra incognita* of theoretical physics. In this note we use anomalous couplings to learn about the six-dimensional superconformal $(0,2)$ models with nonabelian gauge symmetry. Our main result is equation (2.5). As a corollary, we easily recover the formula for black hole entropy in M-theory found in [1]. Our main technique is similar to that used in a recent discussion of the Chern-Simons term of $D = 11$ supergravity in the presence of 5-branes [2]. Our considerations turn out to be useful in explaining the Kaluza-Klein origin of Chern-Simons terms in gauged supergravity. This is the subject of section three. We also comment on a mismatch between anomalies in the AdS/CFT correspondence and its implications.

2. Anomalies of the $(0,2)$ theory

2.1. The anomaly polynomial

The fivebrane of M-theory has chiral world-volume fields which lead to potential anomalies in diffeomorphisms of the five-brane world-volume W_6 as well as in diffeomorphisms which act as $SO(5)$ gauge transformations of the connection on the normal bundle N. The anomaly is determined by an eight-form I_8. There are two obvious contributions to I_8. The chiral world-volume fields of the fivebrane lead to a contribution $I_8^{\rm zm}(Q_5)$. The second contribution arises from anomaly inflow as a result of the coupling [3, 4]

$$\int_{M_{11}} C_3 \wedge I_8^{\rm inf} \tag{2.1}$$

and leads to a contribution $I_8^{\rm inf}(Q_5)$. For a charge $Q_5 = 1$ fivebrane these two contributions do not cancel but rather [5]

$$I_8^{\rm zm}(1) + I_8^{\rm inf}(1) = p_2(N)/24\,. \tag{2.2}$$

In [2] it was pointed out that there is a further contribution to the anomaly which arises from a careful treatment of the Chern-Simons term of $D = 11$ supergravity in the presence of fivebranes. With ρ the integral of a bump form $d\rho$ and $e_3^{(0)}$ [1] related to the global angular form $e_4/2$ by $e_4 = de_3^{(0)}$ (see Appendix A for details) the Chern-Simons term is

$$S'_{CS} = \lim_{\epsilon\to 0} -\frac{2\pi}{6}\int_{M_{11}-D_\epsilon(W_6)} (\not{C}_3 - \sigma_3)\wedge d(\not{C}_3 - \sigma_3)\wedge d(\not{C}_3 - \sigma_3)\,, \tag{2.3}$$

where $\sigma_3 \equiv \rho e_3^{(0)}/2$. Here we have also removed a tubular neighborhood of radius ϵ surrounding the fivebrane. The variation of S'_{CS} may be computed using a result of Bott and Catteneo [6] and leads to a contribution

$$I_8^{\rm CS} = -p_2(N)/24\,, \tag{2.4}$$

which cancels the remaining anomaly.

The cancellation of all anomalies for a charge one fivebrane gives us confidence that the anomalies must also cancel for arbitrary charge Q_5 fivebranes. Unfortunately this is difficult to verify explicitly because the world-volume theory for charge Q_5 is not sufficiently well understood. This theory is a non-Abelian tensor theory with $(0, 2)$ supersymmetry.

Instead, we will assume that the anomalies do in fact cancel for all Q_5 and use this to predict what the anomalies of the non-Abelian $(0, 2)$ theory must be. This is easily done since the bulk supergravity contributions from (2.1) and (2.3) can be computed reliably for $Q_5 > 1$. Note that the contribution to the anomaly from $I_8^{\rm inf}$ is linear in C_3 hence linear in Q_5 while the contribution from $I_8^{\rm CS}$ is cubic in C_3 and hence cubic in Q_5. Thus anomaly cancellation requires that

$$I_8^{\rm zm}(Q_5) = Q_5 I_8^{\rm zm}(1) + (Q_5^3 - Q_5)p_2(N)/24\,, \tag{2.5}$$

where [5]

$$I_8^{\rm zm}(1) = \frac{1}{48}\left[p_2(N) - p_2(TW) + \frac{1}{4}\left(p_1(TW) - p_1(N)\right)^2\right]. \tag{2.6}$$

[1]Throughout this paper we use the notation of the descent formalism, $\omega_n = d\omega_{n-1}^{(0)}$, $\delta\omega_{n-1}^{(0)} = d\omega_{n-2}^{(1)}$ for a closed gauge invariant n-form ω_n

Here TW denotes the tangent bundle to the fivebrane worldvolume W, and N is the normal bundle to W in the bulk M_{11}.

The extension of (2.5) to the D and E series of $(0,2)$ theories and to the nonabelian $(0,1)$ theories remains open.

2.2. Application 1: Correlators in the $(0,2)$ theory

Equation (2.5) contains some nontrivial information about the current correlators of the $(0,2)$ nonabelian superconformal theory of nonabelian tensor-multiplets. This theory has an $OSp(6,2|4)$ superconformal current multiplet $\mathcal{J}$ whose structure is given in part by [7]: $\mathcal{J}_{\rm bosonic} = (J^{(IJ)}, J^{[IJ]}_\mu, J^I_{\mu\nu\lambda}, J_{\mu\nu})$, where $I = 1, \ldots, 5$ is an $so(5) \cong usp(4)$ $\mathcal{R}$-symmetry index in the fundamental of $so(5)$, the scalar term $J^{(IJ)}$ is in the **14**, $J^{[IJ]}_\mu$ are the $\mathcal{R}$-symmetry currents in the adjoint **10**, the anti-selfdual 3-form currents $J^I_{\mu\nu\lambda}$ are in the **5** and the energy-momentum tensor $J_{\mu\nu}$ is a singlet. These currents can be coupled to a contragredient multiplet of background fields $\Phi_{\rm bosonic} = (\pi^{(IJ)}, A^{[IJ]}_\mu, S^I_{\mu\nu\lambda}, h_{\mu\nu})$ to form the generator of current correlators:

$$\exp\Big[-\Gamma[\Phi]\Big] \equiv \Big\langle \exp[\int_{W_6} \Phi\cdot\mathcal{J}]\Big\rangle . \tag{2.7}$$

The result (2.5) for the anomaly polynomial implies that if $\epsilon^{[IJ]}$ is an infinitesimal $so(5)$ transformation then the normalized correlator is

$$\begin{aligned}&\Big\langle\Big\langle (\epsilon^{[IJ]} D_\mu J^{[IJ]\mu}) \exp[\int_{W_6} \Phi\cdot\mathcal{J}]\Big\rangle\Big\rangle = \\ &= \tfrac{Q_5^3 - Q_5}{24} \textstyle\int_{W_6} (p_2)^{(1)}_6(\epsilon, A) + Q_5 \int (I_8^{\rm zm}(1))^{(1)}_6(\epsilon, A, h) .\end{aligned} \tag{2.8}$$

Since

$$p_2^{(0)}(A) = \frac{1}{4}(\frac{1}{2}\omega_3^{(0)}\omega_4 - \omega_7^{(0)}) , \tag{2.9}$$

where $\omega_{2n} = (\frac{i}{2\pi})^n trF^n$ and $\omega^{(0)}_{2n-1} = d^{-1}\omega_{2n}$, we find results for $4, 5, 6$ and 7-point functions of currents.

The result (2.8) implicitly contains a good deal of information about correlators in the $(0,2)$ theory. In the $(0,2)$ theory the $\mathcal{R}$-symmetry anomaly is in the same supermultiplet as the conformal anomaly as can be seen by dimensionally reducing the theory to four dimensions. Thus in principle (2.8) provides an exact prediction of the conformal anomaly of the $(0,2)$ theory after carrying out sufficiently many supersymmetry transformations. The details of this calculation are worth doing, but we have not worked them out. The argument might be similar to the discussion in appendix A of [8]. Nevertheless, without working out the details we may make some qualitative observations. The Q_5^3 dependence of (2.5) is in accord with the expectations from black hole calculations [9] and with a large Q_5 calculation of the conformal anomaly using the AdS/CFT correspondence [10]. The novelty here is that we also predict an exact correction to the leading Q_5^3 dependence which is down by $1/Q_5^2$.

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Equation (2.5) provides some interesting clues to the structure of the still unknown microscopic description of the $(0,2)$ theory. For example, it is quite likely that (2.5) and (2.8) can be used to derive an infinite number of correlation functions when the $(0,2)$ theory is compactified on 6-folds of $SO(4) \times SO(2)$ holonomy. The strategy follows the ideas of [11]. One would start with the group cocycle class associated with (2.5), then twist the theory to produce a scalar supercharge $\mathcal{Q}$ so that some of the currents are $\mathcal{Q}$-exact. The absence of gauge fields contragredient to the $\mathcal{Q}$-exact currents in the effective action then fixes the actual group cocycle representing the class (2.5). This determination of the "trivial cocycle" fixes the kinetic terms in a generalized gauged WZW-type action. The resulting WZW-type action then serves as a generating function for an infinite number of current correlators.

2.3. Application 2: Black hole entropy

A precise check of (2.5) and its connection to the conformal anomaly can be made by reducing the world-volume theory to a $1+1$ dimensional conformal field theory. To do this we consider wrapping a charge Q_5 M-theory fivebrane on a supersymmetric four-cycle P in a Calabi-Yau threefold X.

Let $\{\theta_A\}$ be an integral basis for $H^2(X,Z)$ and denote the dual basis of $H_4(X,Z)$ by $\{\sigma_A\}$. The three-form potential of $D=11$ supergravity reduces to a set of $U(1)$ vector fields through the Ansatz $C_3 = \sum_A C_1^A \wedge \theta_A$. A single fivebrane wrapped on a smooth four-cycle in the homology class $P_0 = P_0^A \sigma_A$ gives rise to a string in $4+1$ dimensions which carries charge P_0^A under the $U(1)$ gauge field C_1^A. In what follows we write C_1 for the linear combination of $U(1)$ gauge fields determined by the element of H^2 dual to P_0. The string has an $SO(3)$ normal bundle and the zero mode and inflow contributions to the anomaly fail to cancel by a term involving $p_1(N)$. As for the fivebrane this anomaly was shown in [2] to be cancelled by a modification of the Chern-Simons term of $D=5$ supergravity.

We now consider how the story changes when we scale the homology class $P_0 \to Q_5 P_0$. Demanding that the anomaly be cancelled for charge Q_5 wrapped fivebranes predicts that the zero mode anomaly is given by

$$I_4^{\rm zm}(Q_5) = Q_5 I_4^{\rm zm}(1) + (Q_5^3 - Q_5) p_1(N) D_0/4\,, \tag{2.10}$$

where [2]

$$I_4^{\rm zm}(1) = \frac{c_2 \cdot P_0}{48}(p_1(TW) + p_1(N)) + \frac{D_0}{4} p_1(N)\,, \tag{2.11}$$

with

$$D_0 \equiv \frac{1}{6}\int_X \hat{P}_0^3 = D_{ABC} P_0^A P_0^B\,, P_0^C \tag{2.12}$$

where $\hat{P}_0$ is the dual to P_0.

The string obtained from the wrapped fivebrane is described at low-energies by a conformal field theory with $(0,4)$ supersymmetry. The right-moving superconformal

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algebra has an $SO(3)$ Kac-Moody algebra whose level k is related to the $SO(3)$ normal bundle anomaly above. Specifically, the level k is equal to the coefficient of $p_1(N)/4$ in (2.10) which gives

$$k = Q_5^3 D_0 + Q_5 c_2 \cdot P_0/12\,. \tag{2.13}$$

The right-moving Virasoro central charge is given in terms of the level of the $SO(3)$ Kac-Moody algebra by $c_R = 6k$ so we see that the conformal anomaly has terms cubic and linear in Q_5.

Let us now compare our result for c_R with the result in [1]. We assume that P_0 is the divisor class of a line bundle $\mathcal{L}_0$ defining an embedding of X into projective space (that is, $\mathcal{L}_0$ is "very ample"). In particular, the linear system $|\mathcal{L}_0|$ has no basepoints (i.e. points where all sections of $\mathcal{L}_0$ vanish). Then, for $Q_5 > 0$ the linear system $|Q_5\mathcal{L}_0|$ also has no basepoints and by Bertini's theorem the homology class Q_5P_0 has a representative by a smooth 4-cycle.[2] If the 5-brane is wrapped on such a smooth 4-cycle the analysis of the zeromodes used in [1] is justified, and leads to exactly the same result, $c_R = 6k$ with k given by (2.13). This confirmation of a known result gives us added confidence in (2.5).

We would like to conclude this section with an observation on black hole entropy and anomaly inflow. As discussed in [1], the microscopic configuration of a fivebrane wrapping the five-cycle $P \times S^1$ and with momentum along the S^1 can be described in a certain regime as an extremal black hole solution of $N = 2$, $d = 4$ supergravity. The entropy can be computed macroscopically in terms of the area of the event horizon of the black hole and including a one-loop topological correction. On the other hand the entropy can also be computed by counting microscopic states in the $(0, 4)$ SCFT of the effective string. This leads to an entropy $S = 2\pi\sqrt{c_L q_0/6}$, where q_0 is the total momentum of the left-moving excitations and c_L is the left-moving Virasoro central charge.

The previous analysis determined c_R by cancellation of the normal bundle anomaly, but $c_L - c_R$ and hence c_L can be determined from cancellation of the tangent bundle anomaly. In particular, the tangent bundle anomaly on the worldsheet is given in terms of $c_R - c_L$ by

$$(c_R - c_L)\left(\frac{p_1(TW)}{24}\right). \tag{2.14}$$

This has to be compared with the anomaly inflow from variation of the term

$$\Delta S^5 = \lim_{\epsilon\to 0}\frac{c_2 \cdot P_0}{48}\int_{M_5 - D_\epsilon(W)} C_1 \wedge p_1(TM)\,. \tag{2.15}$$

coming from reduction of $C_3 \wedge I_8^{\rm inf}$ on the manifold X. We thus see that the cancellation of the tangent bundle anomalies requires $c_L - c_R = Q_5 c_2 \cdot P_0/2$. This again is in agreement with [1], and gives

$$c_L = 6Q_5^3 D_0 + Q_5 c_2 \cdot P_0 \tag{2.16}$$

[2]See [12], p. 137, et. seq. for a discussion of the relevant math.

as expected. Thus both the left and right conformal anomalies of this $(0,4)$ theory and hence the black hole entropy are completely determined by anomaly cancellation. This explains in part the results of [1] which reproduced the black hole entropy precisely without the need for string theory or a complete microscopic description of M theory.

3. Chern-Simons terms in gauged supergravity

Gauged supergravity theories in odd dimensions typically contain Chern-Simons couplings of the gauge fields. These have been found in the literature using the Noether method to determine the supersymmetric completion of the Einstein action. Recently these Chern-Simons terms have been of interest in connection with the AdS/CFT correspondence [13, 14, 15]. Indeed, following [15] we identify the background fields Φ of (2.7) with the "boundary values" of the AdS_7 maximally extended supergravity multiplet and $\Gamma[\Phi]$ as the on-shell supergravity action (suitably regularized). In particular, it follows [15] that the anomaly on the boundary of AdS space associated with the variation of the Chern-Simons terms should match the anomaly computed in the boundary CFT.

Since gauge supergravities arise from compactification of higher-dimensional theories on compact spaces with isometries (typically spheres in the simplest examples) it must be possible to understand the Chern-Simons terms from Kaluza-Klein reduction [16, 17]. Unfortunately, such reductions are notoriously subtle, and to our knowledge have not been carried out in the literature to the non-linear order necessary to see the Chern-Simons terms. We will show here that the formalism necessary for smoothing out brane sources has a direct application to this problem. We then comment on the matching of anomalies.

3.1. Chern-Simons terms in AdS_7

Seven-dimensional supergravity in AdS_7 with gauge group $SO(5)$ arises by Kaluza-Klein compactification of M-theory on S^4 [18]. The vacuum configuration is given by the standard metrics on the maximally symmetric space $AdS_7 \times S^4$ and

$$\not{G}_4 = Q_5 \epsilon_4\,, \tag{3.1}$$

with ϵ_4 the volume form on S^4 and $\not{G}_4 = G_4/2\pi$. In order to carry out a Kaluza-Klein reduction it is necessary to expand the metric and four-form field strength to include fluctuations. This was done at the linearized level in [19], but the extension to the nonlinear theory is not obvious. The required ansatz is highly constrained by the requirements that G_4 be gauge invariant under $SO(5)$ gauge transformations, that the Bianchi identity $dG_4 = 0$ be satisfied, and by the requirement that $\int_{S^4} \not{G}_4/2 = Q_5$. The formalism used in [2] is well-suited to finding such an ansatz.

In the presence of fluctuations of the $SO(5)$ Kaluza-Klein gauge fields the ansatz (3.1) can be made gauge covariant by replacing ordinary derivatives with covariant derivatives but it is then not closed without the addition of extra terms. The modifications needed to make it closed while maintaining gauge invariance were derived in [2] and lead to the ansatz

$$\not{C}_4 = Q_5 e_4(A)/2 + \text{fluctuations} \quad \text{in} \ \ \mathrm{C}_3\,, \tag{3.2}$$

where $e_4(A)$ depends on both the metric and the $SO(5)$ gauge field A. An explicit expression is given in the appendix. Keeping only terms involving the $SO(5)$ gauge fields the Kaluza-Klein reduction of the Chern-Simons term then gives

$$\begin{aligned} -\frac{2\pi}{6}\int_{M_{11}} \not{C}_3 \wedge \not{C}_4 \wedge \not{C}_4 &= -\frac{2\pi Q_5^3}{6}\int_{M_{11}} \frac{e_3^{(0)}}{2} \wedge \frac{e_4}{2} \wedge \frac{e_4}{2} \\ &= -\frac{2\pi Q_5^3}{24}\int_{AdS_7} p_2^{(0)}(A)\,, \end{aligned} \tag{3.3}$$

where in the last step we have used the Bott-Cattaneo formula. Using (2.9) above we may compare with the expressions in the supergravity literature [20] and we find agreement.

In addition to the order Q_5^3 Chern-Simons term (3.3) there must also be additional Chern-Simons terms which are linear in Q_5 and which follow from Kaluza-Klein reduction of the $C_3 \wedge I_8^{\rm inf}$ term of $D = 11$ SUGRA. Note that these terms would not have appeared in earlier treatments because the $C_3 \wedge I_8^{\rm inf}$ term does not mix under supersymmetry with the Einstein term but only with other higher dimension terms in the M theory effective action [21, 22]. If we restrict attention to the leading order in a derivative expansion the Kaluza-Klein reduction is easily carried out as follows. The integration over S^4 requires a factor of the volume form. We extract this term from $e_4(A)$ (the other terms contribute to higher derivative interactions). Next we observe that, *to leading order in the derivative expansion* the spin connection in the standard Kaluza-Klein ansatz, restricted to a cross section of the trivial bundle $AdS_7 \times S^4 \to AdS_7$ is simply a direct sum connection[3]

$$\omega_{AB} = \begin{pmatrix} \omega_{\alpha\beta} & 0 \\ 0 & (\nabla_a K_b^{[IJ]})A^{[IJ]} \end{pmatrix} + \cdots \tag{3.4}$$

on AdS_7 for $T(AdS_7) \oplus E$ where E is a vector bundle associated to the $SO(4)$ tangent space group of S^4, and restricted to AdS_7. In (3.4) the indices A, B are eleven-dimensional tangent space indices, α, β are AdS_7 tangent space indices, and a, b are S^4 tangent space indices. Also, I, J are indices in the fundamental of $SO(5)$ and $K_b^{[IJ]}$ are components of Killing vectors on S^4. Equation (3.4) is an identity for 1-forms on AdS_7.

Now, $I_8^{\rm inf} = (p_2 - p_1^2/4)/48$, so we may use the formula for the total Pontryagin class of a direct sum $p(E \oplus F) = p(E)p(F)$, which is valid at the level of forms for a

[3]See, e.g., equation $(A.21)$ in [17].

direct sum connection, to obtain the Kaluza-Klein reduction on AdS_7 to leading order in derivatives:

$$2\pi Q_5 \int_{AdS_7} \left\{ (I_8^{\rm inf})_7^{(0)}(R) - \frac{1}{48}\left[\frac{p_1^2(A)}{4} - p_2(A) - \frac{p_1(R)p_1(A)}{2}\right]_7^{(0)} + \cdots \right\}, \tag{3.5}$$

where now $p_i(R)$, $p_i(A)$ are representatives of the Pontryagin classes of the seven-dimensional tangent bundle and $SO(5)$ principal bundle respectively. Supergravity in AdS_7 thus contains both the order Q_5^3 Chern-Simons term (3.3) and the order Q_5 terms (3.5).

Similar techniques should allow a direct Kaluza-Klein derivation of Chern-Simons terms in other cases of interest such as IIB SUGRA on $AdS_5 \times S^5$. In this case there are several additional subtleties. These include the lack of a covariant action for the self-dual five-form of IIB theory and the fact that in the Kaluza-Klein reduction the massless $SO(6)$ gauge multiplet is in fact a linear combination of a Kaluza-Klein mode of the ten-dimensional metric and a Kaluza-Klein mode of the IIB 4-form potential [23].

3.2. Implications for the AdS/CFT correspondence

The AdS/CFT correspondence requires matching of the anomalies computed in the CFT and from variation of the Chern-Simons terms in AdS supergravity. This matching has been verified in detail at large N [24].

Since the anomaly is exact, we can also ask whether the anomaly matches at subleading order in N. Consider for example the correspondence between $N = 4$ SYM and supergravity on $AdS_5 \times S^5$. Classical supergravity predicts the Chern-Simons term has coefficient N^2. On the other hand, the exact coefficient can be computed from anomaly inflow arguments. Since the gauge multiplet on the D3 branes is $SU(N)$, and *not* $U(N)$ [15, 25], the exact coefficient of the Chern-Simons term must in fact be proportional to $N^2 - 1$. If the correspondence is correct then there must be a correction to the Chern-Simons term (and by supersymmetry a correction to the Einstein term as well) of order $1/N^2 \sim g_s^2 \alpha'^4$. Such a one-loop higher derivative correction does not seem to be ruled out by any known renormalization theorems. Confirming this correction would be a non-trivial check of the correspondence at finite N and therefore of the correspondence in one-loop string theory and not just in the classical supergravity limit. Similar comments apply to the $AdS_7 \times S^4$ case where again there must be an order $1/N^2$ correction to the leading N^3 behavior (in addition to the order $N = Q_5$ term of (3.5)).

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A. Non-singular branes

Consider a p-brane with worldvolume W_d (the longitudinal coordinates are x^μ, $\mu = 0, 1, \ldots d = p+1$) located at $y^a = 0$, $a = 1, 2, \ldots D-d$ in the total space M_D. The most naive expression for the Bianchi identity in the presence of the brane (the magnetic source equation) is

$$dG_{D-d-1} = 2\pi\delta(y^1)\cdots\delta(y^{D-d})dy^1 \wedge \cdots \wedge dy^{D-d} \, . \tag{A.1}$$

The quantity on the right hand side is a $(D-d)$-form with integral one over the transverse space and delta function support on the brane. In order to have a completely well defined and non-singular prescription in such cases we need to smooth out the delta function source. Having done this we will see that in the presence of a non-zero $SO(D-d)$ connection on the normal bundle we will have to modify the right hand side of (A.1) in order that it transform covariantly under $SO(D-d)$ gauge transformations.

This has been done in [2] for the case of the M-theory fivebrane. After defining a radial direction away from the brane, we cut out a disc of radius ϵ around it. That is, we remove a tubular neighborhood of the brane of radius ϵ, $D_\epsilon(W_d)$. The map $\xi : D_\epsilon \to W_d$ is a fibration with the fiber being an open $(D-d)$-ball which may be considered as the unit ball in the normal bundle. The restriction of ξ to the set of points, $S_\epsilon(W_d)$, whose distance to W_d is fixed (and smaller than ϵ) can be identified with a unit sphere in the normal bundle and thus has fiber S^{D-d-1}. Note that $S_\epsilon(W_d)$ is the boundary of the tubular neighborhood $D_\epsilon(W_d)$.

In order to smooth out the brane source we choose a smooth function of the radial direction with transverse compact support near the brane, $\rho(r)$, with $\rho(r) = -1$ for sufficiently small r and $\rho(r) = 0$ for sufficiently large r. The bump form $d\rho$ then has integral one in the radial direction. The smoothed form of (A.1) should then read

$$dG_{D-d-1} = 2\pi\Phi_{D-d}, \tag{A.2}$$

where Φ_{D-d} represents the Thom class of the normal bundle. One can write this representatitive in terms of the global angular form and ρ. The expression depends on whether the rank of the bundle is even or odd and is given by:

$$\begin{aligned} \Phi_{D-d} &= d\rho \wedge e_{2n}/2, & 2n &= D-d-1 \\ &= d(\rho \wedge e_{2n-1}/2), & 2n-1 &= D-d-1 \, . \end{aligned} \tag{A.3}$$

The global angular form e_{D-d-1} is gauge invariant under $SO(D-d)$ transformations of the normal bundle. Φ_{D-d} should reduce to the naive expression on the r.h.s of (A.1) for a flat infinite fivebrane when $d\rho$ approaches a delta function. Physically what we are doing is smoothing out the magnetic charge of the brane to a sphere of magnetic charge linking the horizon.

We now give explicit formulae for the global angular form on a real vector bundle $E \to M$ with metric and connection. Let E_0 be the complement of the zero-section.

If $\mathrm{rank}(E) = 2n+1$ is odd then the sphere bundle has fibers S^{2n}. The global angular form e_{2n} on E_0 restricting to the volume form on the fibers of $S(E)$ satisfies

$$de_{2n} = 0\,, \tag{A.4}$$

so the Euler class vanishes:[4] $\chi(E) = 0$. Moreover, $e_{2n}/2$ has integral one over the fibers of S_ϵ. The global angular form is given by

$$e_{2n} = \frac{1}{2(4\pi)^n n!}\sum_{j=0}^{n}(-1)^j \frac{n!}{j!(n-j)!}\epsilon_{2n+1}(F)^j(D\hat{y})^{2n-2j}\hat{y}\,, \tag{A.5}$$

where $\hat{y}^{\hat{a}} \equiv y^{\hat{a}}/r$ and are defined only outside of $0 \in \mathbb{R}^{D-d}$ ($\hat{a} = 1, \ldots, D-d$). There is a globally-defined connection Θ on the total space of the $SO(D-d)$ bundle in terms of which we have

$$\begin{aligned} (D\hat{y})^{\hat{a}} &\equiv d\hat{y}^{\hat{a}} - \Theta^{\hat{a}\hat{b}}\hat{y}^{\hat{b}} \\ F^{\hat{a}\hat{b}} &= d\Theta^{\hat{a}\hat{b}} - \Theta^{\hat{a}\hat{c}} \wedge \Theta^{\hat{c}\hat{b}}\,, \end{aligned} \tag{A.6}$$

and

$$\epsilon_{2n+1}(F)^j(D\hat{y})^{2n-2j}\hat{y} \equiv \epsilon_{\hat{a}_1\ldots\hat{a}_{2n+1}}F^{\hat{a}_1\hat{a}_2}\ldots F^{\hat{a}_{2j-1}\hat{a}_{2j}}(D\hat{y})^{\hat{a}_{2j+1}}\ldots(D\hat{y})^{\hat{a}_{2n}}\hat{y}^{\hat{a}_{2n+1}}. \tag{A.7}$$

The cohomology class e_{2n} satisfies [6]

$$[e_{2n}^2] = \pi^*(p_n(E)), \tag{A.8}$$

Moreover, at the level of differential forms we have

$$\pi_*(e_{2n}^3) = \pi_*\left(e_{2n}\pi^* p_n\right) = 2p_n \tag{A.9}$$

for the expression above.

If $\mathrm{rank}(E) = 2n$ is even the sphere bundle has fibers S^{2n-1}. There is a global angular form e_{2n-1} on E_0 restricting to the volume form on the fibers of $S(E)$ such that

$$de_{2n-1} = -\pi^*(\chi(E)) \tag{A.10}$$

for $\chi(E) \in H^{2n}(M;\mathbb{Z})$. The global angular form is given by

$$e_{2n-1} = -\frac{1}{(2\pi)^n}\sum_{j=0}^{n-1}\frac{2^{-j}}{j!(2n-2j-1)!!}\epsilon_{2n}(F)^j(D\hat{y})^{2n-2j-1}\hat{y}\,. \tag{A.11}$$

[4] This equation holds rationally. In fact, one only needs to be able to invert 2.

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